

# Modeling Lake-Groundwater interactions in GFLOW

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## Modeling objectives

Lakes can be included in GFLOW as a closed string of line-sinks with a given (constant) head, that are positioned along the lake shore. If the lake is to be part of a stream network, it would be necessary to use separate line-sink strings along the lake boundary between inlet and outlet streams with a small head gradient to define the direction of flow from inlet stream to outlet stream. This gradient may be very small, but is necessary to assure proper streamflow routing in the network that contains the lake. The head given to the line-sinks is the “lake stage,” which must be known.

Detailed studies of the hydrological interactions between lakes, surface waters and groundwater require that the lake stage be solved for, rather than specified in advance. The purpose is to model the hydrologic regime and establish a detailed water budget for the lake. In fact, the lake stage results from the requirement that the lake is in perfect water balance with the inputs and outputs to the atmosphere, surface waters and groundwaters to which it is connected. There are different approaches to modeling these “lake interactions.” Some researchers envisioned the lake as a zone in the aquifer with a very high hydraulic conductivity (Lee 1996, Hunt and Krohelski, 1996 and Hunt et al., 2000). While such a conceptual model for the lake will indeed produce the lake stage as part of the model output, it does not lend itself to a complete analysis of all components of the lake water balance. Several packages have been developed for MODFLOW to perform such a complete lake water balance analysis: LAK1 (Cheng and Anderson, 1993), LAK2 (Council, 1998) and LAK3 (Merritt and Konikow, 2000). Similar modeling capabilities have been added to GFLOW, by introducing options on the line-sink menu to define “lakes”, “outlet” streams and, for the case of high-k lakes, “inlet” streams. The functionality of these features will be described and illustrated below.

## High-k Lakes

Lakes that are well-connected to the aquifer and which dimensions are large when compared to the aquifer thickness may be conceptualized as areas of little or no resistance to groundwater flow. In a groundwater flow model such a lake can be defined by an area of very high aquifer conductivity in which a nearly horizontal groundwater surface develops: the lake stage. Computationally this appears to be the most efficient approach to modeling a lake. However, the opportunity for a detailed lake water balance analysis is limited.

### Defining the lake and its properties

To enter the lake in GFLOW, simply enter an *inhomogeneity domain* with a high hydraulic conductivity. There is no need to change the bottom elevation. The hydraulic conductivity should be set about three orders of magnitude higher than the surrounding conductivity. For instance, if the surrounding hydraulic conductivity is 75 *ft/day* the inside conductivity may be set to 75,000 *ft/day*. The “lake” defined in this manner will be fully connected to the aquifer, hence it will have no resistance to flow between the lake water body and the groundwater flow regime. If some resistance to flow is desired a *horizontal barrier* may be placed around the outside of the lake to simulate that resistance. The *width* and *conductivity* of the horizontal barrier are obtained from the following analysis.

Assume that the lake bottom has a thickness  $d$  and conductivity  $k_l$ . The resulting lake bottom resistance  $c$  then follows from:

$$c = \frac{d}{k_l} \quad (1)$$

Assume that the aquifer underneath the lake bottom has a thickness  $H$  and conductivity  $k$ . The *characteristic leakage length*  $\lambda$  for the lake is then defined as:

$$\lambda = \sqrt{kHc} \quad (2)$$

The “high-k” lake with bottom resistance will only be computationally robust (always lead to a good solution) if the characteristic leakage length  $\lambda$  is small compared to the smallest lake dimension.

The flow in the aquifer at the lake shore is given by (see “dealing with resistance to flow into surface waters”, Henk Haitjema, equation 12 ):

$$Q_x = \frac{kH\Delta\phi}{\lambda} \tanh(b/\lambda) \quad (3)$$

where  $\Delta\phi$  is the head in the aquifer at the lake shore minus the lake stage and where the parameter  $b$  is the distance from the lake shore to a point inside the lake that is midway between the opposite lake shores. The purpose of  $b$  is to define the extent to which water will enter the lake that crosses the lake shore in question. Note: the analysis is only exact when vertical resistance to flow inside the aquifer can be ignored and the lake is a straight infinitely long water

body of width  $2b$ . For finite and curved lake shores, therefore, equation (3) is approximate.

The barrier around the lake has a thickness  $w$  and a conductivity  $k_b$ . The flow through the barrier is:

$$Q_x = \frac{k_b H \Delta \phi}{w} \quad (4)$$

For values of  $\lambda \ll b$ , to which this analysis is restricted, the hyperbolic tangent becomes 1 and setting (3) equal to (4) yields:

$$k_b = \frac{wk}{\lambda} \quad (5)$$

The barrier width may be selected arbitrarily and  $k_b$  calculated from (5). Conversely, the barrier hydraulic conductivity may be selected, for instance, equal to the surrounding conductivity  $k_b = k$  in which case the width should be selected as  $w = \lambda$ , see (5).

## Defining inlet and outlet streams

On the “Routing” tab of the “Linesink String Properties” dialog the user may select “Use streamflow routing” and after that check “Outlet Stream” for defining an outlet stream and “Inlet Stream” for defining an inlet stream. The *last* linesink of an inlet stream should be placed entirely inside the inhomogeneity that defines the lake (one but last vertex on the inhomogeneity boundary). The *first* linesink of an outlet stream must be placed entirely inside the “lake” (second vertex on the inhomogeneity boundary). In the event that a resistance barrier surrounds the lake, the one but last (inlet stream) or second (outlet stream) linesink must be between the barrier and the inhomogeneity boundary, see Figure 1.

The outlet stream requires the specification of a file with a “stage table”. This file is a user supplied ASCII file, an example of which follows:

```
395  400000
400  500000
405  600000
410  700000
415  800000
420  900000
425  1000000
430  1010000
quit
```

The first column contains stage levels and the second column contains the associated flow rates out of the lake into the outlet stream. Outflow rates will be linearly interpolated between lake stages in the file. Outside the range of these lake stages (lower than 395 or larger than 430) the outflow rate is kept constant (400000 and 1010000, respectively).

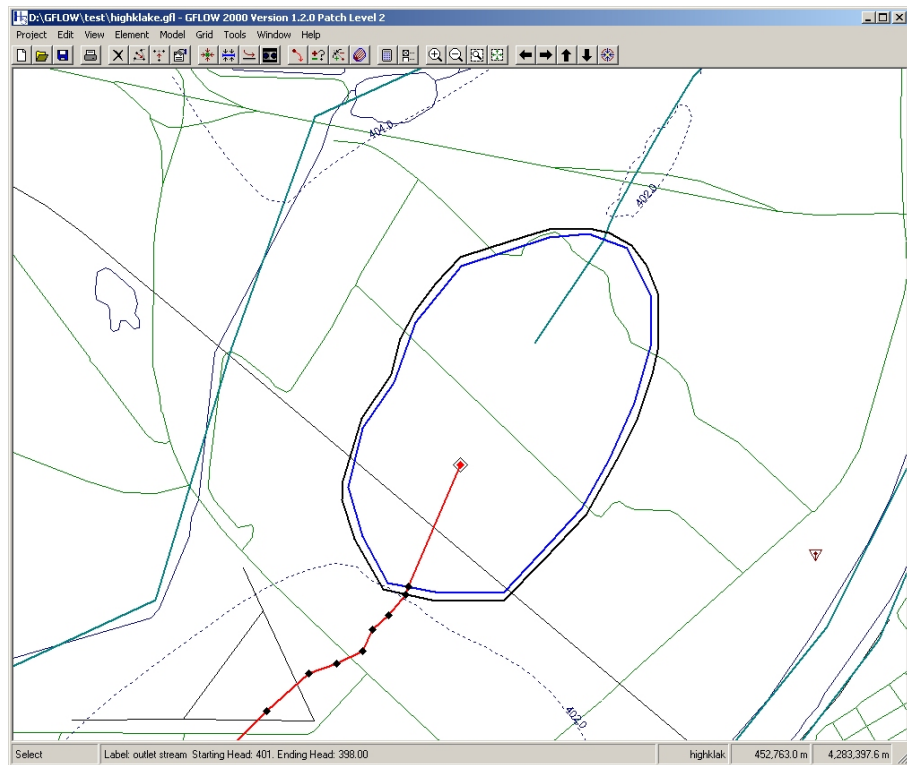


Figure 1: High-k lake with inlet and outlet stream. Note: one line-sink is inside the lake and one line-sink is between the lake and the horizontal barrier used to simulate bottom resistance for the lake (projectfile: highlake.gfl).

Inlet streams and outlet streams can be part of a stream network. Consequently, the outlet stream can be an “End Stream” and form the end of a network or, if not declared an end stream, will be connected to another linesink string that is declared as a stream feature. The inlet stream *must* be declared as an “End Stream” (forced by the GUI) in order to release its water into the lake. The “Inlet Stream” option on the *Routing tab*, when selected, forces all streamflow in the inlet stream to be infiltrated into the aquifer by the last linesink in the string. The “Outlet Stream” option on the *Routing tab*, when selected, forces the first linesink in the string to extract the amount of water from the aquifer as prescribed by the “stage table”. Since both the last linesink of the inlet stream and the first linesink of the outlet stream are located inside the high-k inhomogeneity (the lake), these waters are released into and withdrawn from the lake, respectively.

## Solution strategies and lake water balance

The Solver settings (*Model>Settings>Solver*) are the same as those for a normal model with stream features (stream networks). Select the option “Conjunctive Surface Water - Groundwater Solution” and specify between 6 and 12 iterations depending on the complexity of the groundwater flow model. After the Solver is done, thus after the Dos boxes close and a redraw occurred, you should check the accuracy with which boundary conditions are satisfied by opening the “Runtime Message File” (*Model>View Model Run Files..>Runtime Message File*). Percentage errors should in general be below 1% and are usually orders of magnitude smaller than 1%. These errors only reflect how conditions at collocation points are met, elsewhere along a stream or inhomogeneity boundary the conditions may not be met as accurately. Consequently, you should visually inspect the quality of the solution as exhibited by potentiometric contours. If a resistance barrier is used a jump will occur in the potentiometric contours across that barrier, which leads to a bundling up of contours near the barrier with an often hooky pattern due to the grid used to produce the potentiometric contours.

The inhomogeneity lake should be carefully inspected for a correct water balance. To do so, left-click on the last vertex of inlet streams (should be inside the lake) and verify that the last line-sink inside the lake turns yellow. Next right-click to open the “Linesink Information:” box. Record the “Total Discharge” of the line-sink, which should be negative and reflects the total amount of water that the inlet stream adds to the lake. Do this for all inlet streams. **Important:** Verify that the streamflow is nearly zero at the last vertex of each inlet stream. The remaining streamflow is a water balance error: it is water in the stream that is not injected into the lake or if negative, it represents an over-injection of water. Similarly, left-click on the *second* vertex of an outlet stream (should be on the lake boundary) and verify that the first line-sink in the lake turns yellow. Next right-click to open the “Linesink Information:” box. Record the “Stream flow” for that line-sink, which is the total amount of water which the outlet stream receives from the lake. Repeat this procedure for all

other outlet streams. Next place a flux inspection line around the lake (on top of the lake boundary). Make sure the flux inspection line forms a closed contour. Click on the calculator icon and answer “yes” to use the existing solution. After the DOS box closes you can click on the flux inspection line and read on the status bar the total groundwater flow into or out of the lake. Finally, calculate the total flow into or out of the lake due to recharge. To do this, realize that the recharge rates specified for inhomogeneities are cumulative (labeled as “Added Recharge Rate.”) You must know the area of the lake to calculate this component of the water balance. Note: While the GUI does not report the area for an inhomogeneity, there is a way to find out. Go to *Model>View Model Run Files.>Inquiry File* and scroll down in the file to the point where the inhomogeneities are. The total area for the inhomogeneity is reported under a label “domain area”. After these inspections you know all inflows and outflows for the lake and can check the lake water balance. Realize that you only get the net groundwater inflow or outflow and that precipitation and evapotranspiration are combined into the net recharge rate that the lake receives. The high-k lake also does not allow for an overland flow contribution to the lake. Consequently, not all components of the lake water balance are independently available, but the net inflows and outflows should still balance.

Under certain circumstances the solution may not converge; no water balance is obtained for the lake. This is particularly likely for cases where a resistance barrier is surrounding the lake. It is possible to stabilize the solution procedure for such cases and force convergence, as follows. Add a head specified well inside the lake (roughly in the center). Give the well a head that is the expected lake stage. Solve the model and check the discharge of the well. If the well has a positive discharge, the lake stage (the specified head at the well) is too large. Conversely, if the discharge is negative the head is too low. Make an adjustment to the head at the well and solve again. Through an iterative process of adjusting the head at the well until the discharge is (nearly) zero the proper lake stage (head at the well) can be obtained.

## Line-sink Lakes

As an alternative to the “high-k” lake, a lake may be modeled with a closed string of line-sinks along its shore. These line-sinks can be given a *width* and *resistance* parameter to account for lake bottom resistance. The head along the line-sink string is constant (the lake stage) and will be iteratively determined during the solution process. The lake can be given a precipitation rate and evapotranspiration rate, which will be applied to the lake area. The lake area, as reflected by the line-sink geometry, is not the same as the lake area used in water balance calculations. The latter is defined in a table of lake stages verses lake areas stored in an ASCII file on disk. The Solver (GFLOW1.EXE) will automatically define a negative recharge over the lake to cancel the recharge specified in the area of the lake. This recharge inhomogeneity is exported in the \*.xtr file, but not read back by the GUI (thus not visible in the GUI).

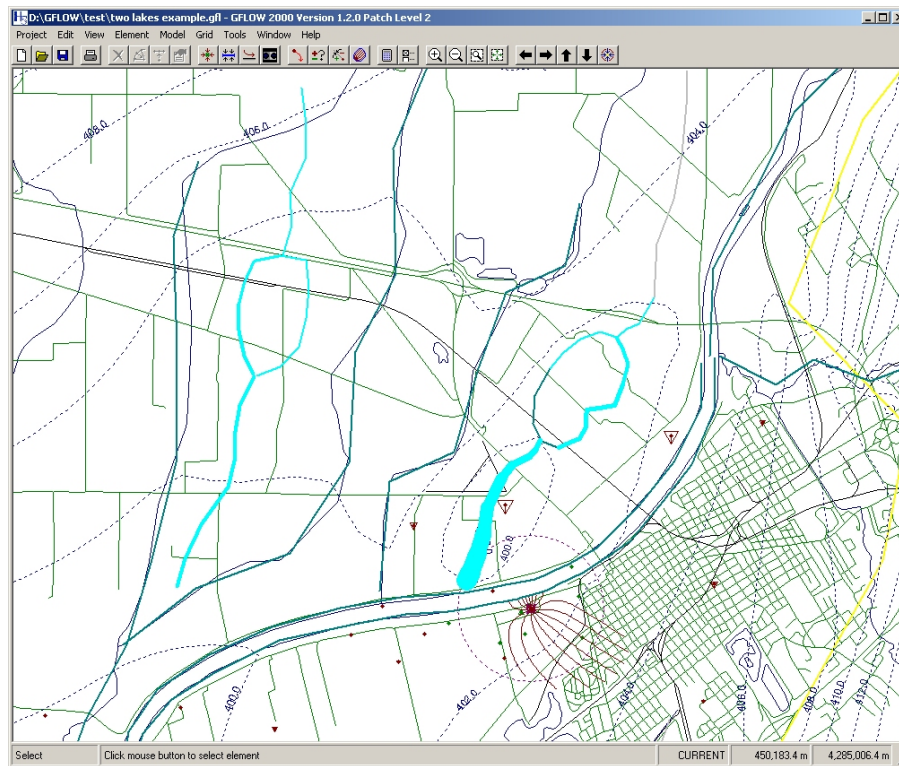


Figure 2: Two line-sink lakes, each with an inlet and outlet stream (project file: two lakes example.gfl).

Inlet streams are regular line-sink strings defined to be a stream feature. Outlet streams are defined with a table that relates the lake stage to the lake outflow rate into the stream. The line-sink lakes will require a longer solution procedure (more iterations) than the high-k lake, but will be stable for all lake bottom resistances and allow for a complete lake water balance analysis. An example of two line-sink lake features, each with an inlet and an outlet stream is shown in Figure 2.

## Defining the lake and its properties

Enter a string of line-sinks along the lake shore. On the **General tab** of the Linesink Properties menu check the “Lake” option, see Figure 3. When checking the “Lake” option the specification of a Starting Head and Ending Head will be replaced by “Estimated Level 1” and “Estimated Level 2”. These two levels are the starting values for a secant method approach to solve for the lake stage. The actual lake stage may be between the estimated levels or higher or lower than these levels. You must also define a “Lake Bottom” elevation, below which

the lake stage cannot drop. The *resistance* and *width* parameters are defined as follows:

Assume that the lake bottom has a thickness  $d$  and conductivity  $k_l$ . The resulting lake bottom resistance  $c$  then follows from:

$$c = \frac{d}{k_l} \quad (6)$$

The resistance  $c$  as calculated from (6) is entered as the “Resistance” on the *General* tab. The “Width”  $w$  should be set to the actual width of the lake. For a circular lake this is simply the diameter, but for an arbitrarily shaped lake this is somewhat problematic as the width varies. For an elongated shape of the lake it is best to select the smallest dimension, excluding one or more local narrow sections. In other words, select a width parameter that is an under bound for most, but not necessarily all line-sinks. The “Linesink Location” must be set to “Along surface water boundary.” The “Depth” parameter should be set equal to the anticipated lake stage minus the elevation of the bottom of the resistance layer underneath the lake. It will force the lake to start percolating when the head in the aquifer drops below the bottom of the resistance layer.

On the **Routing** tab you may specify an “Overland Flow” for the lake, an “Evapotranspiration” rate and a “Precipitation” rate. You should also specify the filename for the “stage table”, which defines the lake area as a function of the lake stage. This is a user supplied ASCII file, an example of which follows:

```
395  24000000
400  24500000
405  25000000
410  25400000
415  25600000
420  25700000
425  25800000
430  25900000
quit
```

The left-hand column contains lake stages and the right-hand column the associated lake areas. The total inflow due to precipitation and total outflow due to evapotranspiration is the specified rate times the area found from the lake stage table using linear interpolation between stages. The lake area below the lowest stage and above the highest stage in the table are kept constant (24000000 and 25900000, respectively).

Note: The GFLOW Solver (GFLOW1.EXE) will automatically assign a “recharge only” inhomogeneity to the lake, coinciding with the line-sink string that defines the lake, and with a recharge rate that cancels the recharge that occurs in the area of the lake. This means that the lake will have a zero net recharge rate, as it should. This “recharge only” inhomogeneity, created by the Solver, will not be visible in the GUI. However, the inhomogeneity will be found in the \*.xtr file written by the solver, see *Model>View Model Run*



**Linesink String Properties** (F1 for Help)

Label

General | Routing | Scenarios

☐ Head-Specified ☐ Treat as "far-field"

☐ Discharge-Specified

☐ Drain

☐ Gallery

☒ Lake

Linesink Location

☐ Unknown

☐ Along stream centerline

☒ Along surface water boundary

Estimated Level 1  feet

Estimated Level 2  feet

Resistance  days

Width  feet

Depth  feet

Figure 3: Linesink String Properties dialog with settings for "lake 1".

*Files.>Inquiry File.* This special inhomogeneity is given GFLOW labels that are the same as those for the line-sinks that define the lake to which it belongs, except that the prefix “LS” is replaced by “IN.”

## Defining inlet and outlet streams

An inlet stream for the lake is a regular line-sink string that is defined as a stream feature (check “Use streamflow routing” on the Routing tab). Do **not** check the “Inlet Stream” option! The latter option is only for use with high-k lakes. The inlet stream line-sink string should end at a vertex on the lake. Do **not** check the “End Stream” option, since you want the stream to connect to the lake.

An outlet stream is a stream feature with the option “Outlet Stream” checked. Its first vertex should coincide (approximately) with a vertex of the lake. You may or may not check “End Stream” depending on whether the outlet stream forms the end of a network or is connected to another stream feature. You must provide a filename for a “stage table”. This file is a user supplied ASCII file, an example of which follows:

```
395  400000
400  500000
405  600000
410  700000
415  800000
420  900000
425  1000000
430  1010000
quit
```

The first column contains stage levels and the second column contains the associated flow rates out of the lake into the outlet stream. Outflow rates will be linearly interpolated between lake stages in the file. Outside these lake stages (lower than 395 or larger than 430) the outflow rate is kept constant (400000 and 1010000, respectively).

## Solution strategies and lake water balance

The lake stage is solved for iteratively by use of a secant method as follows. First a complete conjunctive surface water and groundwater solution is generated using “Estimated level 1”  $\phi_1$  (specified on General tab of the Linesink Properties menu). The resulting water balance deficiency  $\Delta Q_1$  is calculated. Next a solution is generated for “Estimated level 2”  $\phi_2$  and again the resulting water balance deficiency  $\Delta Q_2$  is calculated. A new lake stage estimate  $\phi_3$  is calculated from:

$$\phi_3 = \frac{\phi_1 \Delta Q_2 - \phi_2 \Delta Q_1}{\Delta Q_2 - \Delta Q_1} \quad (7)$$

A new solution is generated with lake stage  $\phi_3$  and the resulting water balance deficiency  $\Delta Q_3$  is calculated. These new values of  $\phi$  and  $\Delta Q$  are used in (7), replacing  $\phi_1$  and  $\Delta Q_1$  or  $\phi_2$  and  $\Delta Q_2$ , to arrive at a new prediction for the lake stage. This procedure is repeated several times to converge on the true value for the lake stage. The user controls this solution procedure from the “Solver” tab (*Model>Settings>Solver*). First specify the number of iterations for the conjunctive surface water and groundwater flow solution as “Number of Iterations (inner loop)”. This setting is usually between 4 and 8 iterations depending on the complexity of the conjunctive groundwater and surface water problem. Next the box in front of “Conjunctive Surface Water - Groundwater Solution” should be checked to implement a conjunctive solution. Check the box in front of “\*Apply outside loop iterations. Number of iterations:” and select between 3 and 6 iterations. This outside loop updates the lake stages by use of a secant method as explained above. After a solution has been obtained you must check the lake water balance in addition to the errors at control points. A lake water balance report is found in the Runtime Message File (*Model>View Model Run Files..>Runtime Message File*), an example of which follows:

```

      Lake water balance:
first GFLOW label: LS_000046_0101
last GFLOW label: LS_000046_1901
      lake stage:  0.4009024D+03
      lake area:  0.2459024D+08
      evap. - precip.:  0.4918048D+05
      groundwater in:  0.4772269D+06
      groundwater out:  0.1398677D+06
      stream flow in:  0.2308589D+06
      stream flow out:  0.5180483D+06
      overland flow in:  0.0000000D+00
water balance deficiency:  0.9893427D+03
water balance deficiency:  0.1397207D+00 %

```

Note: The Solver generates two files with the final lake water balance at the end of the iterative solution process: “lakes.out” and “<basename>.lks”. Both files contain the same information, except that the file “<basename>.lks” is annotated exactly like the lake water balance table in the Runtime Message File as shown above. Furthermore, the “lakes.out” file is overwritten each time a run with lakes is completed in the same working directory, while the file “<basename>.lks” remains untouched as long as the run with the same “base name” (defined on the project settings menu) is not being repeated.

You may also left-click on a lake vertex, next right-click for the “Linesink Information” panel and click on the “Lake Information..” button on the bottom of the panel. This will open a lake water balance report, see example in Figure 4. Note: The Evapotranspiration and Precipitation terms are only known to the GUI, the Solver reads and writes the difference between these values: evapotranspiration - precipitation, see the lake water balance table written in the Runtime Message File (message.log). If the lake water balance errors,

| Lake Information                    |                  |                                      |               |                      |
|-------------------------------------|------------------|--------------------------------------|---------------|----------------------|
| Lake Label:                         |                  | <i>lake 1</i>                        |               |                      |
| First Gflow Label                   | LS_000046_0101   | Error                                | 0.1397207D+00 | %                    |
| Last Gflow Label                    | LS_000046_1901   | Error                                | 0.9893427D+03 | ft <sup>3</sup> /day |
| Lake Stage                          | 0.4009024D+03 ft | Area                                 | 0.2459024D+08 | ft <sup>2</sup>      |
| <b>Inflows</b> ft <sup>3</sup> /day |                  | <b>Outflows</b> ft <sup>3</sup> /day |               |                      |
| Groundwater                         | 0.4772269D+06    | Groundwater                          | 0.1398677D+06 |                      |
| Precipitation                       | 24590.24         | Evapotranspiration                   | 73770.72      |                      |
| Overland Flow                       | 0.0000000D+00    | Overland Flow                        | 0             |                      |
| Stream Flow                         | 0.2308589D+06    | Stream Flow                          | 0.5180483D+06 |                      |
| <b>Total</b>                        | 732676.04        | <b>Total</b>                         | 731686.72     |                      |

Figure 4: Lake Information panel with lake water balance data. The panel is accessed by left-clicking a lake vertex and right-clicking to open the “Linesink Information” panel. Next click on the button labeled “Lake Information..”.

streamflow errors, or errors on boundary conditions are not all small enough, additional iterations are necessary. Implement additional iterations by selecting *Model>Continue Solve* and select the desired number of additional iterations for the inner and outer loop.

If you are making model refinements, requiring a frequent repetition of the solution procedure, you may speed up the Solver by suppressing the error reports during each groundwater and surface water solution. This will only lead to a meaningful shortening of the total solution time for large models. This option is selected on *Model>Settings>Solver* or on *Model>Continue Solve* by checking the box “Suppressing reporting for faster solve.”

## References

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